

# Analysis of Magnetic Field and Force in a Tubular Linear Magnetic Gear with Halbach Permanent-Magnet Arrays

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This paper proposes a tubular linear magnetic gear with Halbach permanent magnet (TLMGHPM) arrays. Based on the principle of magnetic permeability modulation, the formulations for the transmitting force which reflects the main transmitting capability of tubular linear magnetic gear (TLMG) are theoretically deduced from the energy method. In order to produce positive force, the steady working point of TLMG is predetermined elaborately. The features of forces on low-speed and high-speed movers are analyzed.

*Index Terms*—Finite element method (FEM), Halbach PM arrays, linear magnetic gear, magnetic circuit.

## I. INTRODUCTION

THE requirement for high force density linear actuators is currently being met mostly by using intermediate transmission mechanism to transform rotary motion to linear motion. However, apart from the low power transmission density, the reliability and maintenance requirements also become significant issues [1]-[2]. The linear magnetic gear having the tubular topology is an alternative approach which increasingly is considered for a wide variety of applications.

TLMG has some distinct advantages, namely, minimum acoustic noise, free from maintenance, improved reliability, inherent overload protection, etc. It is well known that Halbach PM arrays hold some attractive features [3]. The purpose of this paper is to develop mathematical models [4]-[5] and incorporate Halbach PM arrays into the TLMG.

## II. STRUCTURE AND CONFIGURATION

TLMG can transform force and shift speed between the two movers by the modulation of magnetic field produced by each of the concentric tubular permanent magnet movers by the ferromagnetic rings, which are sandwiched between the two movers, such that the appropriate space harmonic having the same number of poles as the associated permanent magnet mover results.

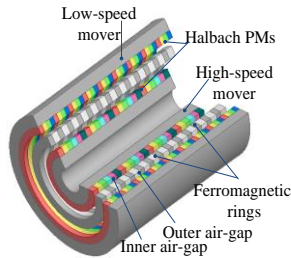


Fig.1. Schematic of TLMGHPM

As shown in Fig.1, the PM pole-pair number of the high-speed mover  $p_h$  is 4, and that of the low-speed mover  $p_l$  equals 9. The number of ferromagnetic segments  $p_s$  is equal to 13, which is the sum of  $p_h$  and  $p_l$ . If the low-speed mover or ferromagnetic rings is held stationary, the speeds of the other parts are governed by gear ratio  $G_r = p_s/p_h$  or  $G_r = -p_l/p_h$

For the proposed topology, the numbers of Halbach PMs per pole are 3 and 2 for the high-speed mover and the low-speed mover, respectively.

## III. MATHEMATICAL MODELING

The model of TLMG in two-dimensional cylindrical polar coordinates is shown in Fig.2. Thereinto,  $z$  denotes the position along the axial direction of the ferromagnetic rings,  $z_1$  and  $z_2$  are the axial positions of PMs on low-speed and high-speed movers with respect to the coordinate origin.

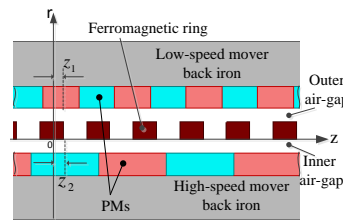


Fig.2. Model of TLMG

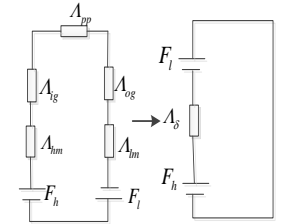


Fig.3. Magnetic circuit model

### A. Magnetic Permeance

In order to simplify the analysis, the following assumptions were made: 1) the end effect of the MG and the flux leakage is negligible; 2) the magnetic field only varies in the radial direction; 3) the permeability of the back irons of two movers and the ferromagnetic rings are infinite; 4) the permeability of permanent magnet is equal to that of air.

Based on the aforementioned assumptions, the magnetic circuit can be considered as linear so that the resultant magnetic field can be treated as the superposition of the fields separately excited by PMs on the two movers.

Fig.3 shows the equivalent magnetic circuit excited by PMs on the high-speed and low-speed movers.  $F_h$  and  $F_l$  denote the magnetomotive forces (MMF) of PMs on the high-speed and low-speed movers. Thus, the equivalent magnetic permeance [6] in the radial direction can be expressed as:

$$\frac{1}{A_\delta(z)} = \frac{1}{A_{hm}} + \frac{1}{A_{lm}} + \frac{1}{A_{ig}} + \frac{1}{A_{og}} + \frac{1}{A_{pp}(z)} \quad (1)$$

where,  $A_{hm}$ ,  $A_{lm}$ ,  $A_{ig}$ ,  $A_{og}$ ,  $A_{pp}$  are the magnetic permeances of the PMs on the high-speed mover, low-speed mover, inner air-gap, outer air-gap and the ferromagnetic pole pieces, respectively.

$A_{pp}$  is a function of the axial position  $z$ . When the segment area is the ferromagnetic pole-piece,  $A_{pp}$  is infinite. On the contrary, the segment is in air space,  $A_{pp} = \mu_0/h_{pp}$ .  $h_{pp}$  is the height of ferromagnetic pole-piece.

Without regard to higher harmonic, the magnetic permeance of the equivalent magnetic circuit of TLMG  $A_s(z)$  is given by:

$$A_s(z) = \lambda_{\delta 0} + \lambda_{\delta 1} \cos\left(p_s \frac{2\pi}{L} z\right) \quad (2)$$

where  $\lambda_{\delta 0}$ ,  $\lambda_{\delta 1}$  are the amplitudes of constant component and fundamental component of the equivalent magnetic Permeance respectively,  $L$  is the active length of TLMG.

### B. Magnetic flux density

Regardless of the high order harmonics, the total magneto motive force can be expressed as:

$$F(z_1, z_2, z) = F_{h1} \cos\left(p_h \frac{2\pi}{L}(z+z_2)\right) + F_{l1} \cos\left(p_l \frac{2\pi}{L}(z+z_1)\right) \quad (3)$$

where  $F_{h1}$  and  $F_{l1}$  are fundamental components of the magneto motive forces of PMs on the high-speed and low-speed movers, respectively. Thus, the magnetic flux density excited by PMs on the high-speed mover can be obtained by:

$$B_{hm} = F_{h1} A_s(z) = F_{h1} \left[ \lambda_{\delta 0} + \lambda_{\delta 1} \cos\left(p_s \frac{2\pi}{L} z\right) \right] \quad (4)$$

### C. Magnetic Energy and Thrust Force

On the basis of electromagnetic field theory, the calculating equation of the magnetic energy stored in air gap can be achieved by:

$$W(z_1, z_2, z) = \frac{1}{2\mu_0} \int_V B^2(z_1, z_2, z) dV = \frac{1}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} F^2(z_1, z_2, z) A_s(z) dz \quad (5)$$

According to the virtual displacement method, the thrust force exerted on the low-speed mover can be obtained by:

$$f_l = \frac{\partial W(z_1, z_2, z)}{\partial z_1} = \frac{1}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\partial F^2(z_1, z_2, z)}{\partial z_1} A_s(z) dz \quad (6)$$

$$= -\frac{\pi p_l F_{h1} F_{l1} A_{\delta 1}}{2} \sin\left(p_h \frac{2\pi}{L} z_2 + p_l \frac{2\pi}{L} z_1\right)$$

A similar expression can be obtained for the thrust force exerted on the high-speed mover.

## IV. STEADY WORKING POINT OF TLMG

In order to produce positive force, the steady working point of TLMG should be controlled properly, which means that the initial positions of the two movers and the ferromagnetic rings should be predetermined elaborately. Based on (6), when the axes of PMs on two movers are consistent with that of ferromagnetic rings, i.e.,  $z_1 = 0$ ,  $z_2 = 0$ , and the polarity of magnets on two movers is opposite, the thrust forces produced by high-speed and low-speed movers equal zero.

## V. PERFORMANCE ANALYSIS

The feasibility of the above theoretical analysis is validated by the simulation analysis based on 2-D FEM.

### A. Magnetic Field Distribution

Fig.4 shows the radial flux density of inner and outer air gaps of TLMGHPM, respectively. The simulation results coincide with the theoretical analysis.

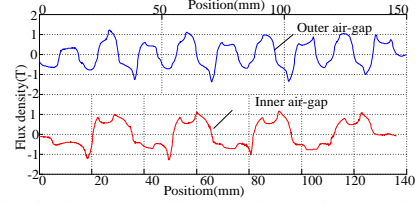


Fig.4. Radial flux density waves of inner and outer air-gaps of TLMGHPM

### B. Static force character Analysis

Fig.5 depicts the static thrust force waveform of TLMGHPM when low-speed mover traveling at the rated speed and ferromagnetic rings and high-speed mover stationary. The corresponding static force character shows the force-time curve varies sinusoidally with time. The force ratio of the low-speed mover with respect to the high-speed mover is 9:4, which is consistent with the fundamental of magnetic gear.

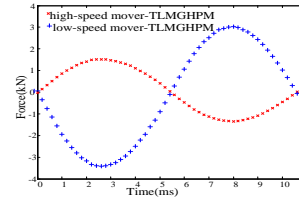


Fig.5. Static thrust force waveform of TLMGHPM

## VI. CONCLUSION

A new tubular linear magnetic gear with Halbach PM arrays has been proposed. By using magnetic circuit modeling, the corresponding air gap flux density and thrust force are analytically derived, which is essential to provide physical insight for machine design. The feasibility of the theoretical analysis is validated by the simulation analysis based on FEM.

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